Online Appendices

Credit Shocks and Equilibrium Dynamics in Consumer Durable Goods Markets

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A Data and Additional Empirical Patterns

In this appendix, we describe in more detail the datasets used in Section 3; we explain the methodology to construct our new- and used-car price indices; we provide additional empirical patterns that complement those of Section 3; and we report some robustness checks about the dynamics of new-car prices.

A.1 Data Sources

In addition to the aggregate data used to construct the annual number of scrapped cars displayed in Figure 2, we use three data sources in Section 3. The first two are rich datasets on new- and used-car prices, respectively. The third is the Consumer Expenditure Survey (U.S. Bureau of Labor Statistics, 2013). We now describe these datasets in more detail.

New-car Prices. This dataset, obtained from Dominion Dealer Solutions (2019), reports the universe of new-vehicle transactions in five states—Colorado, Idaho, North Dakota,

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Ohio, and Texas—for the period 2004-2012, including sales to consumers, leases, and fleet sales. Critically, the dataset reports the transaction price,[A1] the month of the transaction, and the make, model, body, and trim of each vehicle. The dataset includes more than 18 million vehicle transactions.[A2]

Used-car Prices. This dataset, obtained from National Automobile Dealers Association (2016), is an unbalanced panel, reporting historic values of different vintages of vehicle models. It includes two price series, retail and trade-in, for 10 U.S. geographic regions—California, Central, Desert, Eastern, Midwest, Mountain, New England, Northwest, Southeast, and Southwest.[A3] Retail prices represent the "typical selling price" of a transaction between a dealer (as a seller) and a user (as a buyer) for a used vehicle, based on clean conditions; trade-in prices represent the "typical price for a vehicle at trade-in"—that is, a transaction in which a buyer sells an older model to a dealer, using the proceeds as partial payment on a new purchase. NADA updates these used-car prices monthly, based on transaction records at dealerships. We obtained used-car price data for the month of July for every year from 2003 to 2012.

CEX. The CEX is a quarterly survey of U.S. households that, among other things, reports information about households' vehicles at the time of the interview, such as the model, its age, whether it is owned or leased, the acquisition date (although this is often missing), and whether it was acquired new or used.

We use these data from 2003 to 2012 (for comparability with the NADA prices) to compute some aggregate statistics on households' vehicle holdings and transactions. More specifically, the CEX surveys are quarterly, with most households interviewed for four quarters. We define a vehicle replacement when we observe that a household disposes of a vehicle it previously possessed (either owned or leased) and acquires another vehicle, even if these two events happen in different quarters. This definition mechanically implies that households surveyed for fewer quarters are less likely to replace a vehicle than households surveyed for all four quarters. Hence, we restrict our analysis to households surveyed for

[A1] For North Dakota, transaction prices are reported for 2008-2012 only

[A2] We have verified that the number of new-vehicle transactions in the Dominion dataset tracks the aggregate number of new-vehicle sales we plot in Figure 1. This similarity seems to suggest that the Dominion dataset is representative of the entire U.S. market.

[A3] The states included in each region are available at the following link: http://www.nada.com/b2b/Portals/0/assets/pdf/NADA_Regions%20Datasheet_2013.pdf.

at least three quarters and compute our statistics at the annual level.

Although the CEX data are useful for understanding households' decisions regarding their vehicles, we should point out that their use poses some challenges. Most importantly, the sample size of each CEX survey is not large; on average, approximately 7,000 households are surveyed each quarter. Because we further restrict our analysis to households surveyed for at least three quarters, we have approximately 5,600 households per year. Moreover, households trade their vehicles infrequently, which implies that the aggregate statistics we construct based on CEX data are noisy.[A4]

A.2 Construction of New- and Used-vehicle Price Indices

We construct a new-vehicle price index using the Dominion dataset, dropping fleet sales (unfortunately, the price is missing in many of these transactions), and thus exploiting approximately 15.5 million observations on new-vehicle transactions. We estimate the following regression with these data:

$$p^n_{ijst} = \alpha^n_{y(t)} + \beta^n(Year_t - 2004) + \gamma^n_{js} + \epsilon^n_{ijst}, \tag{A1}$$

where the dependent variable p^n_{ijst} is the transaction price of individual vehicle i of make-model-body j (e.g., Toyota Corolla 4L) in state s and month t; $\alpha^n_{y(t)}$ are fixed effects for the year y of the transactions; $(Year_t - 2004)$ is a linear annual trend; γ^n_{js} are fixed effects at the make-model-body j and state s level (e.g., Toyota Corolla 4L in Texas); and ϵ^n_{ijst} are unobservable components of prices. Our new-vehicle price index for year y equals the estimate of $\alpha^n_{y(t)} + \bar{\gamma}^n_{js}$ in equation (A1), i.e., the sum of the year fixed effect and the population-averaged make-model-body-state fixed effects. Hence, our new-vehicle price index holds fixed all car amenities that are common across all vehicles of the same make-model-body-state, thereby varying exclusively due to the year fixed effects $\alpha^n_{y(t)}$ that capture any variation over time within each make-model-body-state and not due to composition changes by which vehicles are sold over time. As a result of our methodology, although we do not have data on repeat sales of the same vehicle i over time and, thus, we cannot

[A4] Households' vehicle sales most likely follow vehicle purchases, rather than vice versa. Hence, our procedure could miss households' replacement when households purchase a vehicle in the last quarter in which they are surveyed (because the subsequent sales are not recorded).

exactly replicate the construction of the Case-Shiller Price index, our price index shares some desirable properties with the Case-Shiller Price index of the housing market.

Similarly, we construct a used-vehicle price index using the NADA dataset. We estimate the following regression on these data:

$$p^u_{ijry} = \alpha^u_y + \beta^u(Year_y - 2004) + \gamma^u_{jr} + \epsilon^u_{ijry}, \tag{A2}$$

where the dependent variable p^u_{ijry} is the NADA trade-in price of a 4-year-old vehicle of trim i of model-model-body j (e.g., Toyota Corolla 4L) in Census region r and year y; α^u_y are fixed effects for the year y of the transactions (we have NADA used prices for the month of July of each year only); $(Year_y - 2004)$ is a linear annual trend; γ^u_{jr} are fixed effects at the make-model-body j and Census region r level (e.g., Toyota Corolla 4L in the Southwestern Region); and ϵ^u_{ijry} are unobservable components of prices. Our used-vehicle price index for year y equals the estimate of $\alpha^u_y + \bar{\gamma}^u_{jr}$ in equation (A2), i.e., the sum of the year fixed effect and the population-averaged make-model-body-region fixed effects. As for the new-vehicle price index, the used-vehicle price index varies over time exclusively due to the year fixed effects α^u_y, and not due to composition changes by which used vehicles trade over time.

Based on the new-vehicle and used-vehicle price indices, we construct a replacement cost index as their difference $\left(\alpha^n_{y(t)} + \bar{\gamma}^n_{js}\right) - \left(\alpha^u_y + \bar{\gamma}^u_{jr}\right)$.

Moreover, we estimate equations (A1) and (A2) separately for three popular models—Toyota Corolla, Honda Civic, and Honda Accord—and construct model-specific new-vehicle, used-vehicle, and replacement cost indices.

Figure 3 displays these replacement costs, normalized to equal 100 in 2007. Figure 4 displays its two components: the new-vehicle price index $\alpha^n_{y(t)} + \bar{\gamma}^n_{js}$ and the used-vehicle price index $\alpha^u_y + \bar{\gamma}^u_{jr}$, normalized to equal 100 in 2007.

A.3 Additional Empirical Patterns

(4) The decline in used-car prices was due to a decline in their demand.

We use the CEX data to investigate the behavior of households in secondary car markets, which can shed light on the decline in used-car prices documented in Figure 4. To this end,

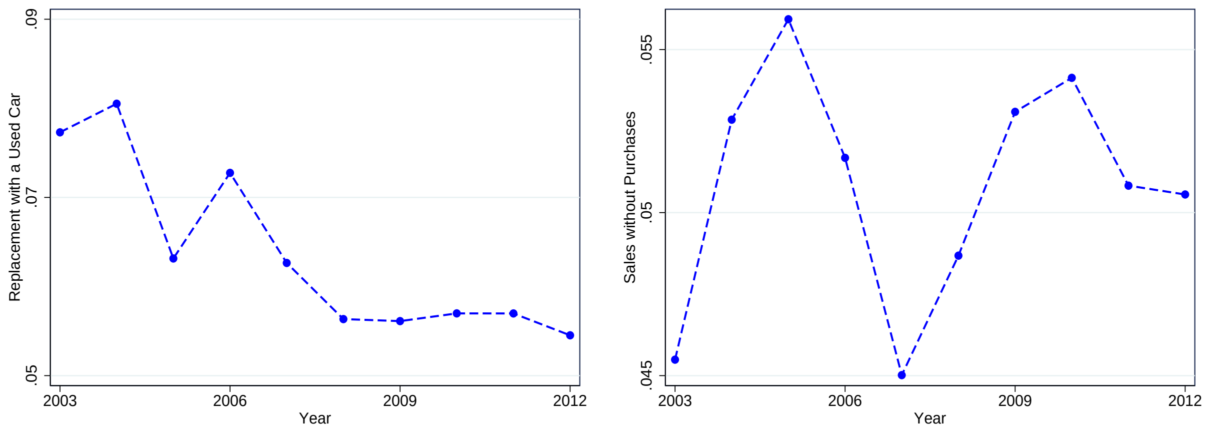

Figure A1: The left panel displays the fraction of households with cars that replaced at least one used vehicle with another used one during 2003-2012. The right panel displays the fraction of households with cars that disposed of a used vehicle without acquiring another one during 2003-2012.

we calculate the fraction of households that replaced a used, old car with another used, but younger, car. The left panel of Figure A1 shows that this fraction *declined* during the Great Recession, thereby suggesting that a decline in the demand for used cars was the main reason for the decline in used-car prices, rather than an increase in their supply.[A5] The right panel of Figure A1 further reinforces the idea that the increase in the supply of used cars during the Great Recession was likely modest, by displaying the fraction of households that sold cars but did not simultaneously purchase another one. Although this fraction increased during the Great Recession, the magnitude of the overall increase from 2007 to 2010 was modest (approximately 0.9 percentage points), and thus it is smaller than the decline in replacement purchases documented in the left panel of Figure A1; more generally, the level it reached during those years was lower than the level it reached pre-2008, whereas replacement purchases clearly bottomed out during the Great Recession.

(5) The average age of registered vehicles increased during the Great Recession.

Consistent with the decline in scrappage and new-vehicle registrations we document, we also observe a steep increase in the average age of the stock of registered vehicles. Figure A2 shows the time series of the growth rate of the average age of all light vehicles in operation.

[A5] Figure 5 and the left panel of Figure A1 together suggest that the decline in used-car prices did not trigger a substitution from new cars to lightly used (i.e., pre-owned) vehicles.

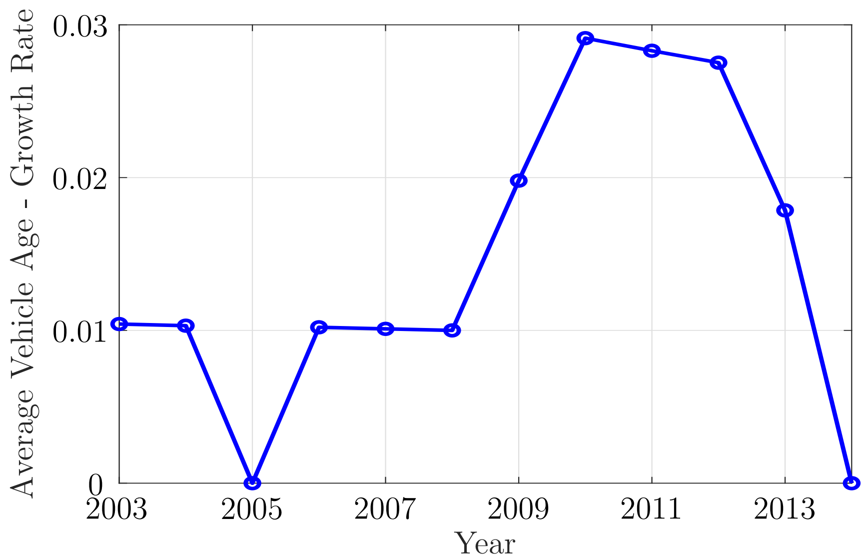

Figure A2: Growth rate of the average age of registered vehicles in the U.S., between 2003 and 2014.

The source of these data is the R.L. Polk Co.

Before the Great Recession, the average age of vehicles was increasing by approximately 1 percent per year, largely because of the effects of technological progress on vehicle durability. In response to the Great Recession, the average age of the vehicle stock increases more rapidly, by approximately 3 percent per year, reflecting the endogenous postponement of new sales and scrappage.

6) Consumer substitution to cheaper new vehicles was limited.

A natural question is to what extent households substituted to cheaper new vehicles during the Great Recession. Our new-car data are well suited for this purpose. To do so, we use these new-vehicle prices to perform a regression similar to (A1), but we include a set of fixed effects γ_s^n at the state s level only rather than the richer set of fixed effects γ_{js}^n at the make-model-body j and state s level. Based on this regression, we construct a different annual new-vehicle price index from the one obtained above: By not including the fixed effects at the make-model-body level, this index does vary over time also because of composition changes by which vehicles are purchased over time.

Figure A3 compares the new-vehicle price index (solid line) portrayed in the top-left panel of Figure 4 with the different new-vehicle price index that also accounts for compositional changes (dashed line). The comparison between the two indices suggests that households substituted to cheaper vehicles during the Great Recession, consistent with the

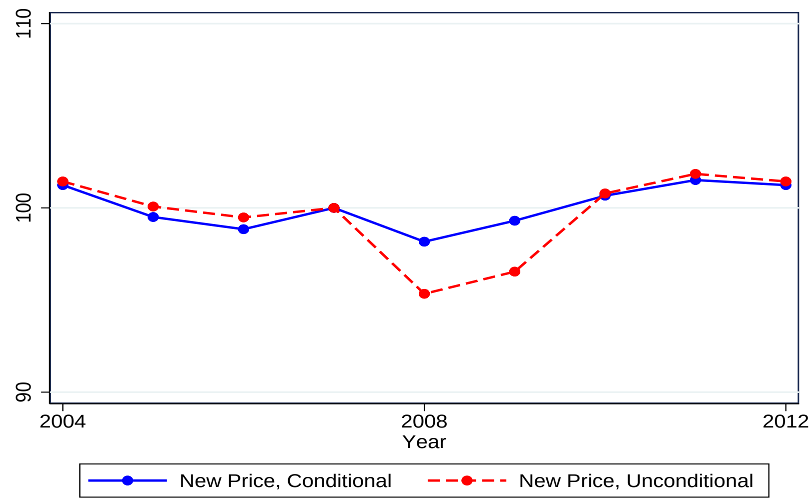

Figure A3: The figure displays new-vehicle price indices: The one represented by the solid line holds compositional changes in new-vehicle purchases constant; the dashed line allows for compositional changes in new-vehicle purchases. Both series are normalized to equal 100 in 2007.

evidence from nondurable goods and services in Jaimovich, Rebelo, and Wong (2019).

Alternatively, we can perform an Oaxaca-Blinder decomposition of new-car prices to understand the extent of consumer substitution towards cheaper new vehicles during the Financial Crisis (Blinder, 1973; Oaxaca, 1973). Specifically, we use our data on new-car transaction prices, we detrend them using a linear trend (as in regression (A1)), and run the following regression separately in the baseline year $t = 2007$ and in the recession year $t = 2008$:

$$p^n_{ijst} = \psi^n_{js,t} \mathbb{1}_{ijst}(js) + \epsilon^n_{ijst}, \tag{A3}$$

where p^n_{ijst} is the detrended (=year 2004) transaction price of individual vehicle i of make-model-body j (e.g., Honda Accord Sedan 4D) in state s and year t; $\mathbb{1}_{ijst}(js)$ are indicator variables equal to one if vehicle i is of make-model-body j and the transaction is in state s, and zero otherwise; $\psi^n_{js,t}$ are the coefficients (i.e., fixed effects at the make-model-body j and state s estimated in year t); and ϵ^n_{ijst} are unobservable components of prices.

Based on these two regressions for the baseline year $t = 2007$ and for the recession year

Table A1: Oaxaca-Blinder Decomposition

$\bar{p}^n_{2007}$	26,426
$\bar{p}^n_{2008}$	25,114
$\psi^n_{js,2007}\left(\mathbb{1}_{ijs,2007}\left(js\right) - \mathbb{1}_{ijs,2007}\left(js\right)\right)$	828
$\left(\psi^n_{js,2007} - \psi^n_{js,2008}\right)\mathbb{1}_{ijs,2008}\left(js\right)$	484

Notes: This table reports the values of the Oaxaca-Blinder decomposition of the difference between the average new-car transaction price $\bar{p}^n_{2007}$ in 2007 and $\bar{p}^n_{2008}$ in 2008. The term $\psi^n_{js,2007}\left(\mathbb{1}_{ijs,2007}(js) - \mathbb{1}_{ijs,2008}(js)\right)$ is the explained component; the term $\left(\psi^n_{js,2007} - \psi^n_{js,2008}\right)\mathbb{1}_{ijs,2008}(js)$ is the unexplained component.

$t = 2008$, the Oaxaca-Blinder decomposition equals:

$$\begin{aligned}\bar{p}^n_{2007} - \bar{p}^n_{2008} = \; & \psi^n_{js,2007}\left(\mathbb{1}_{ijs,2007}(js) - \mathbb{1}_{ijs,2008}(js)\right) \\ & + \left(\psi^n_{js,2007} - \psi^n_{js,2008}\right)\mathbb{1}_{ijs,2008}(js),\end{aligned}$$

where the first term after the equality equals the explained component and the second term equals the unexplained component.

Table A1 reports the numerical values. The overall difference $\bar{p}^n_{2007} - \bar{p}^n_{2008}$ equals approximately $\$1,300$, or about five percent of 2007 prices. The explained component is due to compositional changes in the car purchased and it equals \$848, or approximately three percent of 2007 prices; the unexplained component equals \$484, or approximately two percent of 2007 prices.

Hence, the Oaxaca-Blinder decomposition reports magnitudes almost identical to those displayed in Figure A3, which corroborates that our methodology above yields reliable price indices.

(7) The Consumer Price Index confirms our price patterns.

Figure A4 verifies that the monthly Consumer Price Index (CPI) of new and used vehicles follows patterns similar to those reported in Section 3. It shows that the used-vehicle price deflator declined substantially more than the new-vehicle price deflator during 2007-2009.

We should point out that the CPI indices do not allow for clean comparisons at the

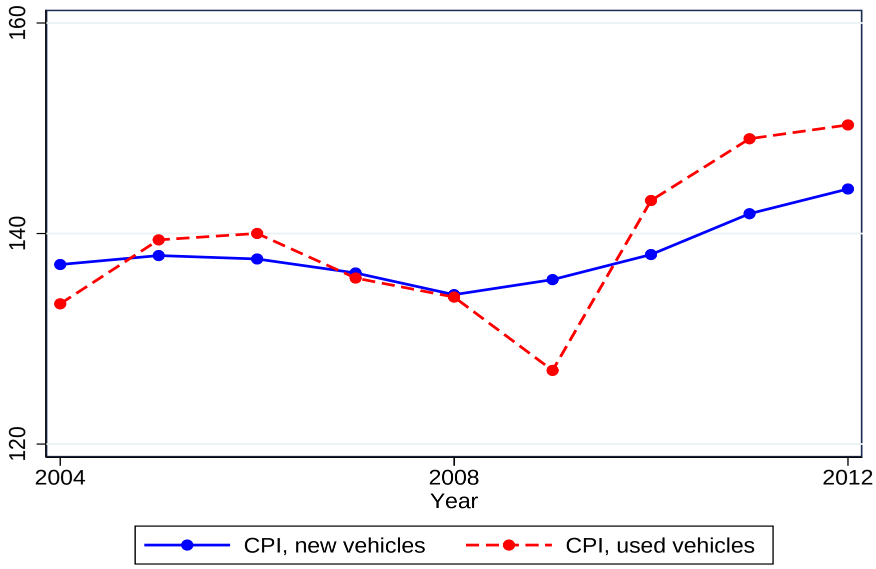

Figure A4: Monthly CPI of new and used vehicles computed by the Bureau of Labor Statistics during 2004-2012.

vehicle level, thus possibly confounding interpretation of the differences between the dynamics of the new and used series. With this important caveat, we can also investigate the dynamics of used-car prices in previous business cycles, which we cannot rely on our transaction data for. We find that in the 2001 recession no drop in used prices occurred, consistent with the notion that the 2001 recession was less associated with a household credit crunch. Going further back in time, we use the whole 1953-2018 BLS sample, and, in the interest of a comparison with the analysis in our micro data, we analyze the data at annual frequency. We thus HP-filter the price indices using a smoothing parameter $\lambda = 6.25$. Consistent with our key empirical findings, the price index for used vehicles is substantially more volatile than the price index for new vehicles. The standard deviation of the cyclical component of used prices equals 0.037, whereas the standard deviation of the cyclical component of new prices equals 0.012.

(8) Manufacturers' cash rebates were limited during the Great Recession.

We complement our analysis of new-car prices with a dataset on cash rebates offered by car manufacturers on purchases of new vehicles. These rebates were advertised in the specialized magazine *Ward's AutoWorld*.[A6] We find that despite some fluctuations over time, these rebates did not increase substantially during the Great Recession.

Specifically, Figure A5 displays the average manufacturer rebate on a new Toyota Camry, one of the popular car models we analyzed in Section 3, during the period 2006-2011. We focus on the Toyota Camry because we have consistent and large availability of data on rebates over time.[A7] The figure shows that the rebate exhibits limited variation over time. While the rebate is larger in 2009 than in 2007, rebates were even larger in both 2006 and 2011—i.e., 2 years of economic expansion. Moreover, the overall variation in the rebates shown in the figure is small relatively to the price of a new Toyota Camry—i.e., between 1 and 3 percentage points of the overall price—and substantially smaller than the volatility of used transaction prices discussed in Section 3.

While these data on manufacturer rebates seem to confirm the robustness of our finding that new-car prices did not change as much as used-car prices during the Great Recession, we should acknowledge that we do not have data on dealer incentives, such as "free gas" or satellite radio membership. The value of these dealer incentives is often quite limited compared to the magnitude of the decline in used-car prices, but we cannot rule out that dealers added such amenities during the financial crisis to a greater extent than in non-recession years. Therefore, the next subsection reports on robustness checks that hone in on these unobservable car amenities.

A.4 Robustness Checks: New-Car Price Index

The goal of this subsection is to report on some robustness checks about our new-car price index. Although our new-car transaction data described above are rich, we acknowledge that they do not report all car characteristics, including dealer incentives. Nevertheless, we can use these data to assess the importance of unobservable car amenities and their variation over the years. Our analysis leverages the key insight of the influential paper by Altonji, Elder, and Taber (2005), which points out that the amount of selection on the observed explanatory variables provides a guide to the amount of selection on the

[A6]We are grateful to Charles Murry for graciously sharing these data with us.

[A7]We construct the series in the figure by averaging over geographic locations. We find a similar pattern if we focus on single trims of this car or we average across trims.

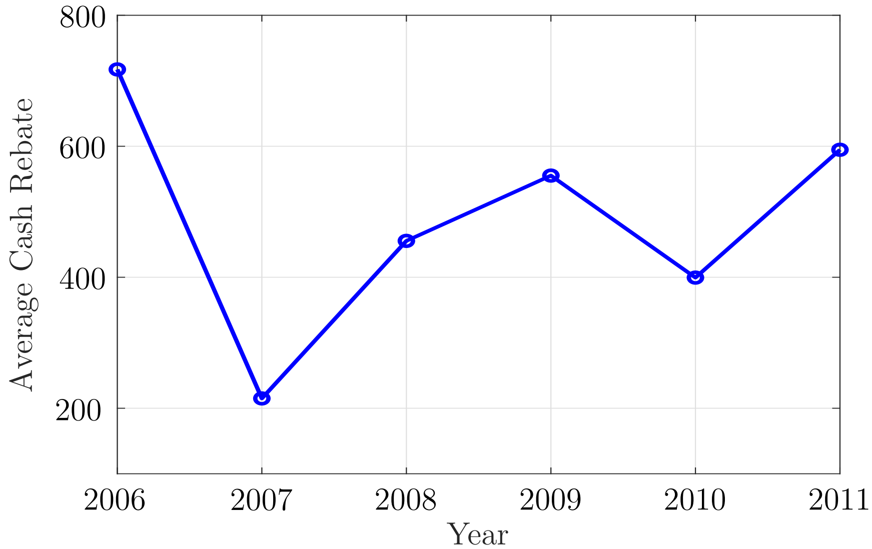

Figure A5: Average cash rebate (in dollars) offered by Toyota on the purchase of a new Toyota Camry between 2006 and 2011.

unobservables.[A8]

More specifically, we perform several regressions in which the dependent variable is the new-vehicle price, and we gradually increase the set of explanatory variables. Comparing the R^2 of these regressions allows us to assess how much these observable characteristics account for the observed variation of prices. More critically, comparing the resulting year fixed effects based on these regressions helps us understand how selection due to compositional changes of car sales over the years affect our new-car index.

Table A2 reports the R^2 and the estimates of the year fixed effects of these regressions. Column (1) documents that the time trend and fixed effects at the make-model-body-state capture approximately 80 percent of the overall sample variation of prices. Specification (2) adds year fixed effects (our main regression equation (A1)), but the R^2 barely changes, suggesting that cyclical variation in prices is minimal. Specification (3) includes fixed effects at the make-model-body-trim-state level, which add two percent to the R^2. Specification (4) includes fixed effects at the make-model-body-trim-dealer-state level, which further increase the R^2 by approximately four percent.

The estimates of the year fixed effects are very similar across specifications (2)-(4). To further appreciate the differences, Figure A6 displays the estimates of the new-price

[A8] Please see Altonji, Elder, and Taber (2005) for the specific conditions required by their methodology.

Table A2: New-Car Prices

	(1)	(2)	(3)	(4)
Time Trend	608.838	579.694	533.569	515.224
	(0.816)	(0.972)	(1.179)	(1.200)
Year 2005		-454.512	-457.708	-413.368
		(6.197)	(6.089)	(5.749)
Year 2006		-628.303	-588.280	-479.761
		(6.072)	(6.224)	(5.959)
Year 2007		-324.621	-478.320	-402.151
		(6.023)	(6.410)	(6.186)
Year 2008		-806.102	-981.379	-905.051
		(6.338)	(6.881)	(6.654)
Year 2009		-507.351	-690.261	-672.646
		(7.076)	(7.495)	(7.187)
Year 2010		-150.990	-295.855	-316.485
		(6.870)	(7.094)	(6.771)
Year 2011		71.780	-73.062	-93.225
		(6.709)	(6.665)	(6.275)
Constant	26165.708	26583.467	26835.983	26872.597
	(3.511)	(5.021)	(5.541)	(5.507)
Make-Model-Body-State FE	Yes	Yes	No	No
Make-Model-Body-Trim-State FE	No	No	Yes	No
Make-Model-Body-Trim-Dealer-State FE	No	No	No	Yes
R^2	0.8068	0.8071	0.8269	0.8689
Observations	15,750,272	15,750,272	15,750,272	15,750,272

Notes: The dependent variable is the transaction price of a new car. Specification (2) corresponds to equation (A1).

indices based on these regressions: the solid line is our main new-car price index based on specification (2), displayed in Figure 4; the dashed line is based on specification (4). The solid and the dashed lines are almost identical.

The regressions of Table A2 underscore two main points: 1) Specification (3) controls for trim characteristics, in addition to make-model-body-state fixed effects of specification (2). However, we note that the increase in the R^2 from column (2) to (3) is small. The monetary values of different trims within make-body-model tend to be larger than the monetary values of dealer incentives, which offer vehicle accessories that can be installed at the dealership. Critically, the addition of these trim controls does not affect the estimates

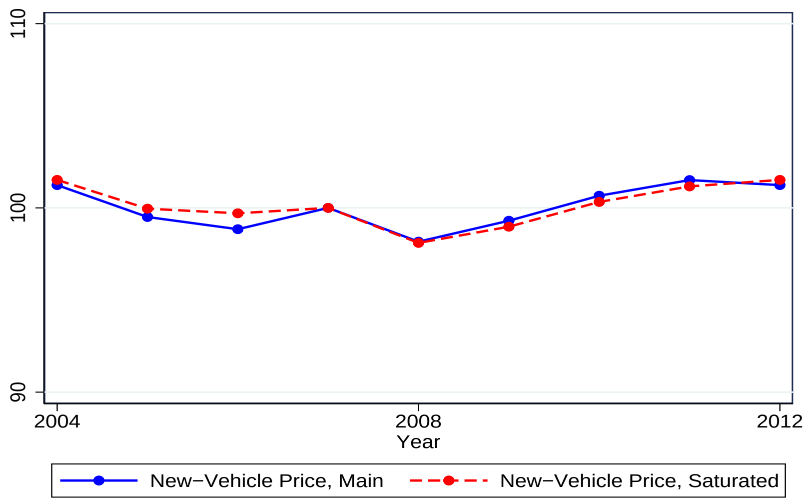

Figure A6: The figure displays new-vehicle price indices: The one represented by the solid line is our main index, based on specification (2) of Table A2; the dashed line is based on specification (4) of Table A2. Both series are normalized to equal 100 in 2007.

of the year fixed effects, implying that selection due to variation in the composition of trims sold over the business cycle does not affect our new-car price index. 2) Specifications (3) and (4) show that further controlling for dealer fixed effects captures some variation in prices, because the R^2 increases by approximately four percent, but this increase is limited as well. However, strikingly, this variation is orthogonal to the year fixed effects because it does not affect their estimates. Hence, selection due to the composition of sales across dealers over the business cycle does not affect our new-car price index.

To summarize, selection due to compositional changes of sales across trims and across dealers over the years does not affect our new-car price indices, as Figure A6 shows. Hence, if selection on these observed explanatory variables provides a guide to the amount of selection on unobservable dealer incentives, as Altonji, Elder, and Taber (2005) postulate, the variation of dealer incentives across years seems unlikely to overturn our key finding that the drop in new-car prices was smaller than the drop in used-car prices during the Great Recession.

B Additional Model Results

In this appendix, we report additional results from our model. First, we derive a general notion of user cost of durable goods in our model. Second, we generalize our framework to consider the case of perfect substitutability in preferences for durable goods. These derivations provide further intuition on key aspects of our mechanism. Third, we consider three alternative formulations of the credit shock, thereby complementing the results of Sections 6.1 and 6.2. Fourth, we decompose the role of equilibrium in secondary markets and the role of transaction costs in the model with both credit and aggregate income shocks of Section 6.3. Finally, we show that the key mechanism highlighted in the paper does not stem from credit-supply shocks only, but, more generally, from shocks that affect the wealth-income distribution asymmetrically.

B.1 User Cost

We now discuss a user-cost interpretation of our results. The user cost of a durable good is the cost associated with enjoying an additional unit of a durable good for only one period. In models that abstract from changes in the prices of durable goods, the user cost is simply the sum of the interest rate (or opportunity cost of capital) and the depreciation rate. In our model, we can similarly define the user cost of an upgrade from a car of quality q_2 to a car of quality q_1 as the replacement cost paid in the current period, net of the discounted revenue from doing the opposite trade (downgrade from q_1 to q_2) in the following period.

With our notation, and focusing for simplicity only on proportional transaction costs (λ), the user cost υ of upgrading from q_2 to q_1 is

$$\upsilon \equiv p_1 - p_2(1-\lambda) - \frac{1}{1+r}\left[(1-\pi_1)p_1(1-\lambda) + \pi_1 p_2'(1-\lambda) - p_2'\right], \tag{B1}$$

where we use primes to denote future-period variables. The formula shows that the user cost can increase because of an increase in the interest rate, as in standard models, or because of a decline in the price p_2. Thus, we can use the formula to quantify the importance of a decline in used-car prices, expressing it in terms of an equivalent change in the interest rate, using our calibrated parameter values and equilibrium prices.

Specifically, we now compute the counterfactual increase in r that would give the same increase in υ in two alternative scenarios. First, we consider a permanent 1-percent decline in p_2. This decline has the same effect on the user cost as a 37-basis-points increase in the interest rate. Second, we consider a 1-percent decline in p_2, but no change in p_2'. In this case, the temporary decline in the used-car price is equivalent (in terms of its effect on the user cost) to a 212-basis-point increase in the interest rate. This analysis provides an alternative interpretation of our key results, by showing that a current decline in used-car prices, combined with expectations of a recovery, has a large effect on the user cost, which induces households to postpone replacement.

B.2 Perfect Substitutability in Preferences

We now investigate the generality of our results in a model with perfect substitutability between durable goods of different qualities, and without indivisibility. This version of the model allows for a first-order condition, or "asset-pricing" equation, for durable goods.

We generalize the household problem in our model to add a continuous choice over durable size, on top of the discrete choice over durable quality, and assume different qualities are perfect substitutes. Moreover, we abstract from transaction costs. For simplicity, we also abstract from ex ante heterogeneity in preferences, i.e., types θ, as well as government debt and taxes, but these features can be included easily. We consider both a small-open-economy and a general-equilibrium version of this model.

Parts of this analysis build on, and adapt to our context, some insights that Rampini (2019) obtains in a model with financially constrained entrepreneurs and new and used capital, assumed to be perfect substitutes in production.

Environment and Household Problem. Households have preferences represented by the following utility function:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, d_{it}), \tag{B2}$$

with

$$d_{it} \equiv \sum_{n=1}^{N} q_n \tilde{s}_{n,it}, \tag{B3}$$

where q_n is durable-good quality, in a discrete set $n = 1, .., N$, and $\tilde{s}_{n,it} \in S_n \subseteq \mathbb{R}_{\geq 0}$ is durable-good quantity or size (or any other continuous attribute) chosen for the current period.

The household budget constraint in stationary equilibrium reads

$$c_{it} + p_b b_{i,t+1} + \sum_{n=1}^{N} p_n \tilde{s}_{n,it} = w_{it} + b_{it} + \sum_{n=1}^{N} p_n s_{n,it}, \tag{B4}$$

where $s_{n,it}$ are beginning-of-period durables quantities.

We impose the following constraints on durable size: $S_n \equiv \mathbb{R}_{\geq 0}$ for all n, and denote by $\nu_{n,it}$ the Lagrange multiplier on the non-negativity constraint for durables of quality n. Our baseline model with indivisibility is a special case of the general formulation above, under alternative restrictions on the choice sets S_n; specifically, $S_n \equiv \{0, 1\}$, and if $\tilde{s}_{m,it} = 1$, then $\tilde{s}_{n,it} = 0$ for $n \neq m$.

As in our baseline model, the borrowing limit is $b_{i,t+1} \geq \phi$, and we denote by $\nu_{b,it}$ the associated Lagrange multiplier.

By taking the first-order conditions of the household problem, we obtain the following "asset-pricing" optimality condition for durables of quality n:

$$p_n u_{ct} = q_n u_{dt} + \beta \left[(1 - \pi_n) p_n + \pi_n p_{n+1}\right] \mathrm{E}_t u_{c,t+1} + \nu_{nt}, \tag{B5}$$

where we drop the dependence of allocations on i to simplify notation, and use u_{ct} and u_{dt} as shorthand notation for the marginal utility from nondurable and durable goods, respectively, at time t.

Equation (B5) explicitly highlights some key features of our theory: (i) heterogeneous durable-good quality affects the "dividend" term from enjoying the durable good; and (ii) expectations about equilibrium resale prices play an important role in the decision to purchase durable goods.[B1]

The first-order condition with respect to bonds is

$$p_b u_{ct} = \beta \mathrm{E}_t u_{c,t+1} + \nu_{bt}. \tag{B6}$$

[B1] For convenience, we assume that each infinitesimal unit of durable good of quality n depreciates with probability π_n. Thus, by the law of large numbers, households know with certainty what fraction of their continuous amount of each quality $\tilde{s}_{nt}$ is going to depreciate. This feature is different from our baseline model with indivisibility, where depreciation is necessarily stochastic.

We now make some additional simplifying assumptions in order to obtain a transparent analytical characterization. Specifically, we assume that only two qualities of cars are useful, i.e., $q_1 > q_2 > 0$. We refer to quality-q_1 cars as new cars and quality-q_2 as used cars. New cars are produced with a linear technology with constant marginal cost p_1 using nondurable goods and become used (q_2) in the next period with certainty. Used cars become worthless scrap ($q_3 = p_3 = 0$) in the following period with certainty. Thus, durable goods are useful for two periods, as new and then as used.

In this case, we can express equation (B5) as follows for qualities q_1 and q_2, respectively:

$$p_1 u_{ct} = q_1 u_{dt} + \beta p_2 \mathrm{E}_t u_{c,t+1} + \nu_{1t}, \tag{B7}$$

$$p_2 u_{ct} = q_2 u_{dt} + \nu_{2t}. \tag{B8}$$

Importantly, these optimality conditions highlight the fact that durables differ not only in their quality, or current utility flow, but also in their future residual value, or durability. In particular, quality and durability are positively correlated.

Equilibrium Definition and Characterization. The state variables of a household are its level of wealth $a_t \equiv b_t + p_2 s_{2t}$ (the sum of bonds and durable goods holdings, because of the absence of transaction costs) and its income w_t. We use the notation $g_n(a, w)$ to denote the size $\tilde{s}_n$ of a durable good of quality n demanded by a household with states (a, w). As in the rest of the paper, we use g_b to denote the policy function for bonds.

The definition of stationary equilibrium is standard: (i) household decision rules for consumption, durable quality and size, and bonds, as a function of the state, (ii) a distribution of households $m(a, w)$, (iii) prices p_b and p_2, such that household decisions satisfy the optimality conditions above, the distribution of households perpetuates itself, and markets clear.

Specifically, in the small-open-economy case, we take p_b as exogenous and only impose the market-clearing condition for used cars:

$$\int g_2(a, w) dm(a, w) = \int g_1(a, w) dm(a, w), \tag{B9}$$

where the left-hand side reports total demand for used cars and the right-hand side reports total supply of used cars, which equals the total amount of new cars depreciated from the previous period.

In the general-equilibrium case, we also impose market clearing in the bond market:

$$\int g_b(a,w)dm(a,w) = 0. \tag{B10}$$

We emphasize that equation (B9) crucially distinguishes this model from models with a homogeneous durable good, which would instead feature the bond market-clearing condition (B10) only or, equivalently, a joint resource constraint for durables and nondurables.

Consider the case in which the marginal household, indifferent between new and used cars ($\nu_{1t} = \nu_{2t} = 0$), is unconstrained in its borrowing ($\nu_{bt} = 0$) (we focus on this case for simplicity; if the marginal household is constrained, our findings discussed below, in particular the endogeneity of prices with respect to the borrowing limit, are even more relevant). The Euler equations above become

$$\frac{p_1}{q_1} = \frac{u_d}{u_c} + p_b \frac{p_2}{q_1}, \tag{B11}$$

$$\frac{p_2}{q_2} = \frac{u_d}{u_c}. \tag{B12}$$

By substituting out the marginal rate of substitution between nondurable and durable consumption, we obtain

$$\frac{p_1}{p_2} = \frac{q_1}{q_2} + p_b. \tag{B13}$$

Equation (B13) showcases that the relative price of new versus used cars depends both on their relative quality and on their relative durability and thus on the interest rate.

At this relative price, all unconstrained households are indifferent between new and used cars. A marginal increase in purchases of used cars and the associated decrease in purchases of new cars can be exactly offset by increasing savings in bonds, leaving the household problem unchanged.[B2]

We now consider the choice of borrowing-constrained households. Combining equations

[B2] Equation (B13) also gives the lower bound for the relative price of used cars in a stationary equilibrium. At a lower p_2, all households would prefer used cars, which is inconsistent with market clearing. In a related model, Rampini (2019) also characterizes equilibria in which used durables trade at a premium, and the marginal buyer is constrained.

(B7) and (B8), we can write

$$\frac{p_1}{q_1} - \frac{p_2}{q_2} = \beta \frac{p_2}{q_1} \frac{\mathrm{E}u_c'}{u_c} + \frac{\nu_1}{q_1 u_c} - \frac{\nu_2}{q_2 u_c}. \tag{B14}$$

Equations (B11) and (B12) jointly imply

$$\frac{p_1}{q_1} - \frac{p_2}{q_2} = p_b \frac{p_2}{q_1}. \tag{B15}$$

Hence, the left-hand side of (B14) is positive. Moreover, for a constrained household, $\beta \frac{p_2}{q_1} \frac{\mathrm{E}u_c'}{u_c} < p_b \frac{p_2}{q_1}$. Thus, we conclude that $\nu_1 > 0$; that is, constrained households only purchase used cars, because they discount the future resale value of new cars at a higher rate than the market interest rate.

Consistent with our baseline model, we obtain that low-wealth households demand cars of lower quality. Furthermore, if the utility function is Cobb-Douglas in nondurables and durables, as in our model parameterization, constrained households demand nondurable goods and used cars in fixed proportions, implying positive comovement between their available resources (income and debt issuance) and their demand for used cars.

We now discuss the extent to which preferences with perfect substitutability pin down equilibrium prices for durable goods. Equation (B13) suggests that in a small open economy (i.e., with an exogenous bond price p_b), the price of used cars is indeed independent of the distribution of households, as long as the marginal household is unconstrained.

In general equilibrium, however, any aggregate change that affects the equilibrium bond price, such as a change in the borrowing limit, also implies a change in the relative price of used cars. The reason is that despite "static" perfect substitutability in utility, new and used cars differ in their durability, with new cars featuring higher durability than used cars (as well as quality). Hence, new cars act as a savings device because they have a positive residual value, whereas used cars have no future resale value.[B3] Thus, no-arbitrage conditions between bonds, new cars, and used cars imply that changes in the price of used cars must offset any changes in the interest rate to ensure that unconstrained households are indifferent between new and used cars, and the market for used cars clears.

[B3] With the general stochastic depreciation structure of our baseline model, used cars also have a positive residual value. Nevertheless, the relative price of used cars still depends on the equilibrium interest rate, because of different expected durability of new and used cars.

Credit Shocks and Durable Goods Markets. We now analyze the transmission of shocks that affect demand for used durables to production of new durables. Consider an economy in stationary equilibrium, and assume that an unexpected shock hits a positive measure of constrained households, by tightening their borrowing limit, only for one period. For simplicity, future borrowing limits are unchanged for all households. Thus, the optimization problem of unconstrained households is unchanged, unless prices change.

Households hit by this temporary credit shock decrease their demand for both nondurable and durable goods (in equal proportions if utility is Cobb-Douglas). Because these households purchase used cars only, total demand for used cars falls.

The transmission mechanism of this shock to other households depends on whether the interest rate is exogenously fixed or endogenously determined. In the small-open-economy version of the model, the bond price is unaffected. Clearly, total supply of used cars is predetermined. Thus, unconstrained households, who are indifferent between new and used cars, partly substitute away from new cars and toward used cars to make up for the demand shortfall, without changing their overall consumption of durables or their total savings. This substitution toward used cars ensures that the used-car market clears (at the same initial price). Overall, the demand shock in the used-car market translates directly to the production of new cars, which must decrease. However, this small-open-economy model cannot account for our empirical findings on the relative price of new and used cars, different from our baseline model.

Moreover, this substitution away from new cars on the part of wealthy households also implies that they increase their bond holdings to maintain the same level of savings. Hence, in the general-equilibrium version of the model, the interest rate needs to fall to clear the bond market. In turn, this interest-rate change implies that the price of used cars must also decrease relative to the initial equilibrium, in order to restore indifference for unconstrained households and ensure market clearing for used cars.

Thus, this model features a version of our novel transmission mechanism: Credit-constrained households decrease their demand for used cars; used-car prices endogenously decline; an increase in the relative price of new cars (relative to used ones) reduces the desirability of new cars for wealthy households.

To highlight the importance of imposing market clearing in the used-car market for this result, consider a counterfactual in which used cars can be re-transformed into nondurables at constant rate p_2^{-1}, as in our counterfactual with fixed prices of Section 6.2. Now, starting from the same initial allocation described above, when the credit shock induces a decline in the interest rate to clear the bond market, the discounted resale value of new cars increases, and all unconstrained households strictly prefer new cars to used cars, thus hitting the non-negativity constraint for used cars, $\nu_{2t>0}$. As all households decrease their demand for used cars, this violates equation (B9), and some used cars are indeed re-transformed into nondurables.

Summary of Results. To summarize this analysis, a version of our model with perfect substitutability in preferences is consistent with several of our findings. In particular, an equilibrium assignment of lower-quality cars to poorer households occurs. As a consequence, a credit tightening manifests itself as a negative demand shock for used cars, and secondary markets transmit this shock to new-car purchases.

In a small-open-economy version of the model, the shock is transmitted directly to wealthy households, but it does not generate endogenous price changes, such as those observed in the data. In general-equilibrium, the model features a version of our novel transmission mechanism through endogenous price changes.

We find that this model does not feature exogenously constant prices for durables, because it imposes market clearing for used cars. Hence, this model does not collapse to the homogenous-durable-good model that is typically analyzed in the literature.

B.3 Alternative Parameterizations of the Credit Shock

We now consider three alternative parameterizations of the credit shock and investigate the sensitivity of our main results. First, we assume the shock is not fully permanent. Second, we consider two alternative calibrations of the shock, matching different credit variables rather than the real interest rate: In one calibration, we match the dynamics of auto loans around the Great Recession; in the other, we match the dynamics of overall household debt.

A persistent, but not permanent, credit shock. Our baseline credit shock is permanent, as in Guerrieri and Lorenzoni (2017). As a consequence, our counterfactuals of Section 6.2 feature different terminal conditions for each path considered, depending on which aggregate series is fed into the household problem or which market clears. We now

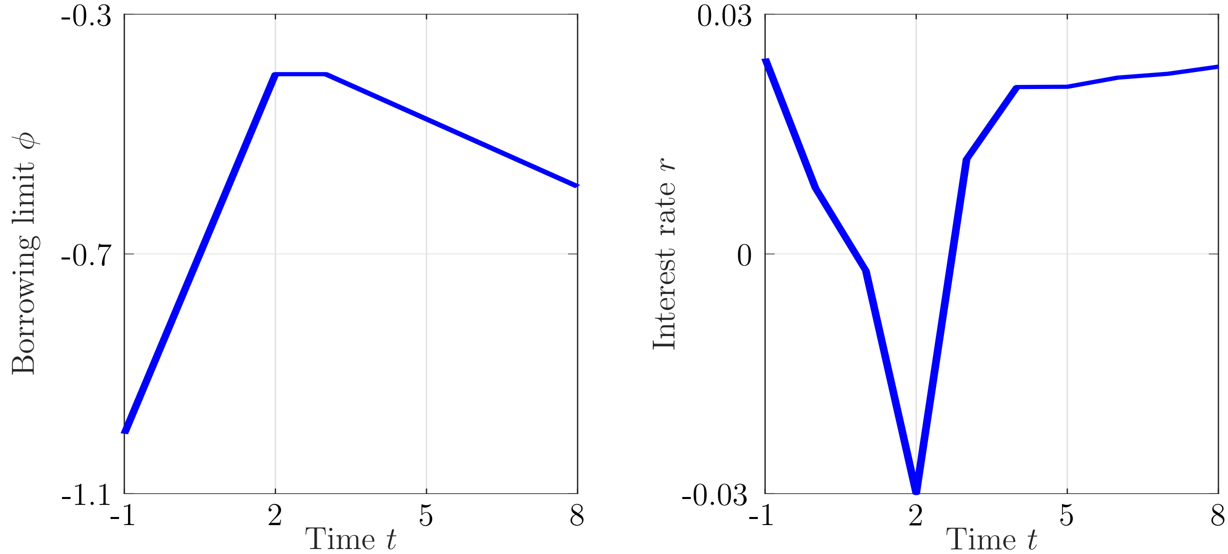

Figure B1: Credit variables with a persistent, but not permanent, shock. The left panel displays the dynamics of the borrowing constraint and the right panel the dynamics of the interest rate. The economy is in the stationary equilibrium at $t = -1$, and households learn about the new path of the borrowing limit at $t = 0$. The borrowing limit reverts to its initial value following a linear path between $t = 4$ and $t = 19$. The horizontal axis displays time t.

show that our results are robust to a change in the assumption on the long-run value of the borrowing limit. To this end, we assume that the borrowing limit eventually reverts to its initial value. We then use this version of our model to follow two alternative calibration strategies.

We assume that the credit shock follows the same path as in our baseline until $t = 3$. Afterward, the shock linearly reverts back to the initial stationary-equilibrium value, reaching it after 15 periods. Thus, the credit tightening is not permanent, but it is highly persistent. In Figures B1 and B2, we report the dynamics of credit-market variables and durable-good market variables, respectively. The equilibrium dynamics induced by this shock are remarkably similar to those of the baseline case considered in Section 6.2. In Figure B3, we repeat the key decomposition of Figure 9 by feeding the dynamics of borrowing limit, interest rate, and used-car price, one at a time, into the household problem. In this case, each series converges to the same stationary equilibrium—that is, the initial one—as shocks and prices eventually return to their initial equilibrium value. Thus, different long-run terminal conditions do not affect short-run differences. We find that our main insights are robust to this modification. Moreover, when the shock is not permanent, the credit tightening would not, *by itself*, trigger a decline in new-car sales. Thus, secondary markets account for the

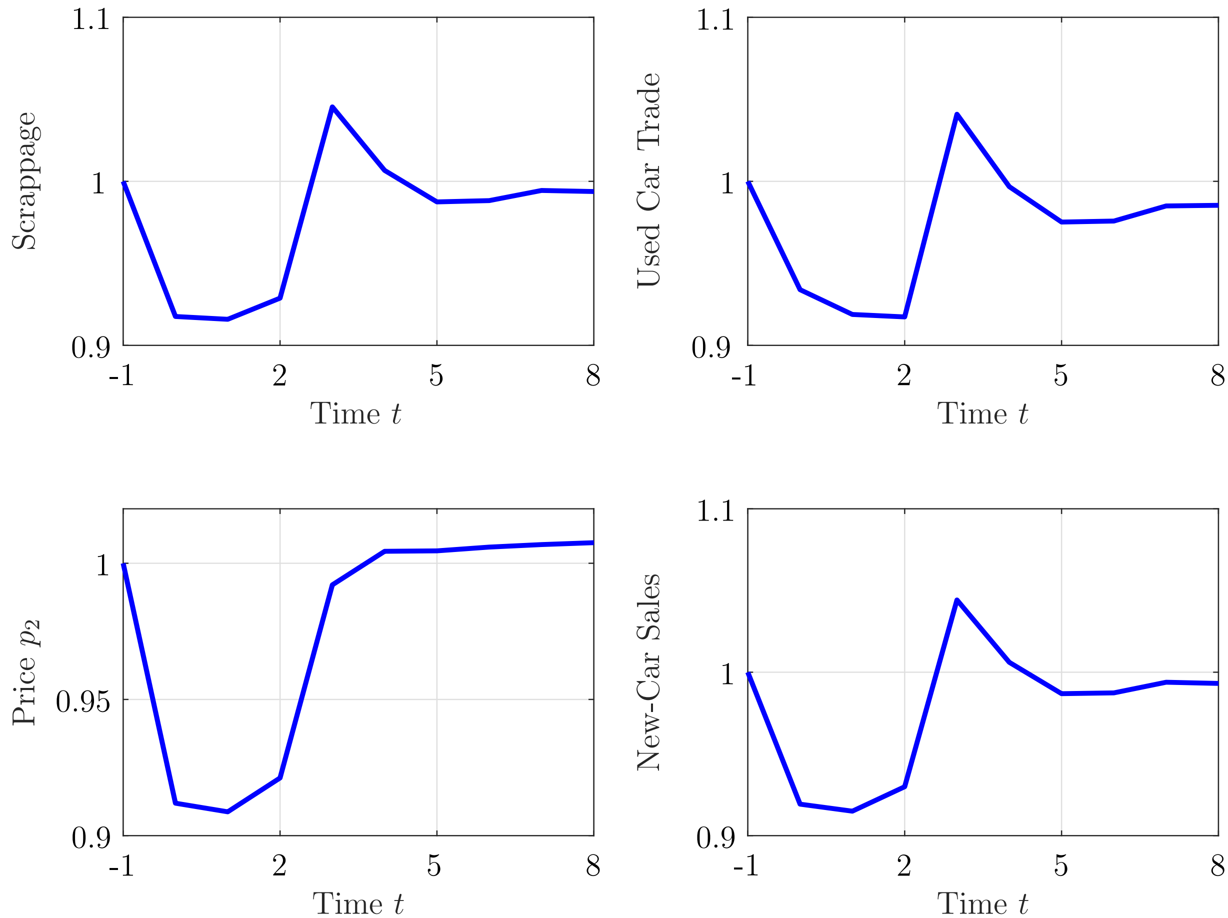

Figure B2: Equilibrium dynamics with a persistent, but not permanent, credit shock. The economy is in the stationary equilibrium at $t = -1$, and households learn about the new path of the borrowing limit at $t = 0$. The horizontal axis displays time t. The top-left panel displays scrappage; the top-right panel the volume of trade of used cars; the bottom-left panel the price p_2 of quality-q_2 cars; and the bottom-right panel the sales of new cars.

entirety of the decline in new-car sales, making our mechanism even more powerful than in the baseline case.[B4]

Alternative calibration 1: Matching auto loans. Our baseline calibration target for the credit shock is the real interest rate. We now consider an alternative approach by targeting credit quantities. First, we use the small-open-economy version of our model and parameterize the credit shock to match the dynamics of auto loans, thus abstracting from interest-rate changes. Specifically, the Consumer Financial Protection Bureau reports data

[B4] We also considered a credit shock that reverts to its initial value more quickly. Small changes in the speed of reversion do not affect our results. When the reversion is very fast, however, we find that the real interest rate features strong nonmonotone dynamics while reverting to its initial value, which appears inconsistent with the fact that the real interest rate has remained low since the Great Recession.

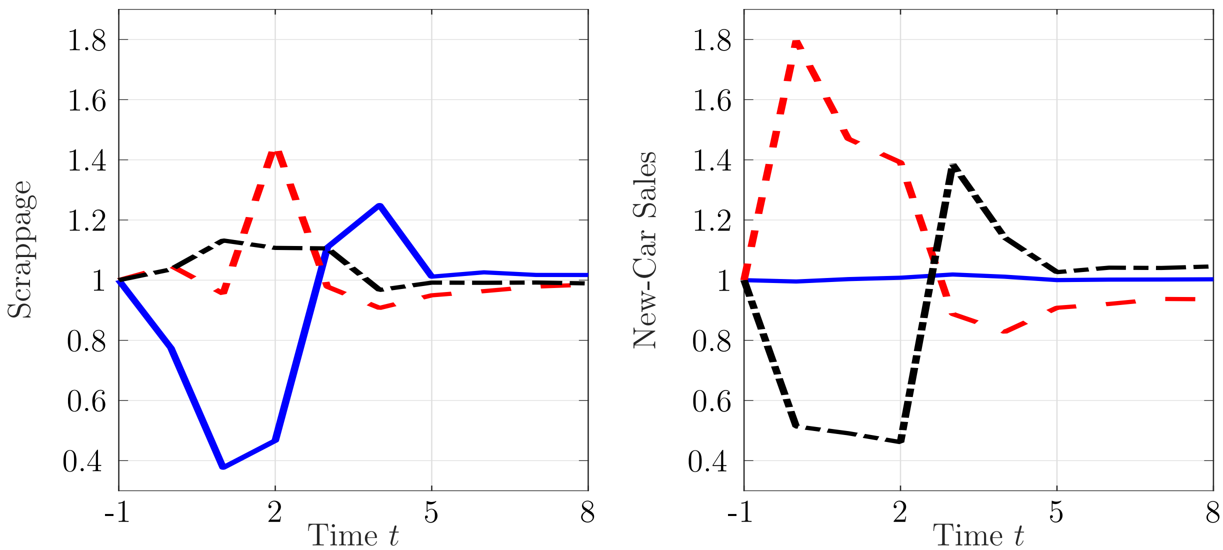

Figure B3: Decomposition with a persistent, but not permanent shock. The left panel displays scrappage and the right panel sales of new cars. The solid line refers to the effects of changes in the borrowing limit; the dashed line to the effects of equilibrium interest rate dynamics; the dashed-dotted line to the effects of equilibrium used-car price dynamics.

on the number and total volume of auto loans originated in each month from 2005. We compute the peak-to-trough decline in these two series during the Great Recession and find that both of them declined by approximately 40 percent. Next, we parameterize the credit shock in our small-open-economy model to match this target, interpreting all debt in our model as auto loans (we discuss an alternative interpretation below). This calls for a slightly smaller tightening than our baseline, namely from $\phi = -1$ to $\phi = -.5$ over 3 years, after which we assume the shock slowly reverts back over 15 years. Under this calibration, the shock induces a 25-percent decline in new-car sales and a 4-percent decline in used-car prices. We see this as further evidence that our key mechanism is also empirically important in the context of a small open economy, with a narrow focus on car markets.

Alternative calibration 2: Matching total household debt. We now consider an alternative notion of credit quantity by mapping debt in our model to a broad notion of household debt in the data. Specifically, we exploit data reported by the Federal Reserve Bank of New York on outstanding household debt between 2003 and 2018 and calculate that the (linearly detrended) stock of household debt declined by approximately 20 percent between 2008 (peak) and 2015 (trough). Accordingly, we target these deleveraging dynamics in our small-open-economy model. This target calls for both a smaller and a more gradual shock in our model. Specifically, the credit limit goes from $\phi = -1$ to $\phi = -.77$

over 7 years, before gradually reverting back over a period of 15 years.

Qualitatively, the results are consistent with the main mechanism of the paper. Scrappage, used prices, and new sales all decline. Quantitatively, however, we find that the gradualism of this shock induces smaller, although more persistent, changes in both prices (approximately 0.5 percent) and quantities (approximately 5 percent). We do not see this version of the model as the most promising to quantitatively account for the main facts on car markets, because we believe that the Great Recession hit car markets more suddenly than what the overall process of household deleveraging over several years implies. This is especially true in the context of a model with perfect foresight, in which most agents can easily plan their response to this projected path of the debt limit. Moreover, we restrict attention to one-period debt in the model. Hence, it is likely that the same target for the reduction in the stock of debt implies a significantly smaller shock on the flow of debt than what would be consistent with the prevalence of longer maturities, as in the data. Thus, we see this as a highly conservative lower bound on the importance of our mechanism. To further corroborate this view, we solved this version of the model in general equilibrium and found that the interest rate decline implied by this shock equals only 0.5 percent.

B.4 Credit and Income Shocks: Inspecting the Mechanism

We study the separate roles of used durable prices and transaction costs in the economy hit by a credit tightening and an aggregate income shock, as in Section 6.3. The results of this decomposition are very similar to our findings in the presence of a credit shock only (Section 6.2), thereby emphasizing that accounting for equilibrium in secondary markets is crucial even in the presence of aggregate income changes.

First, we recompute the transitional dynamics assuming that the secondary market does not clear, and cars trade at their initial prices. Figure B4 displays the resulting equilibrium dynamics. As we found in the case of a credit shock only, scrappage declines substantially and new-car production increases in response to the shocks. Hence, equilibrium in the secondary market is necessary to induce a fall in new car sales, consistent with the evidence from the Great Recession. Relative to Figure 10, the aggregate income shock further decreases scrappage and dampens the initial increase in car sales, which peak 3 years after the initial shocks at over 50 percent above the steady-state value.

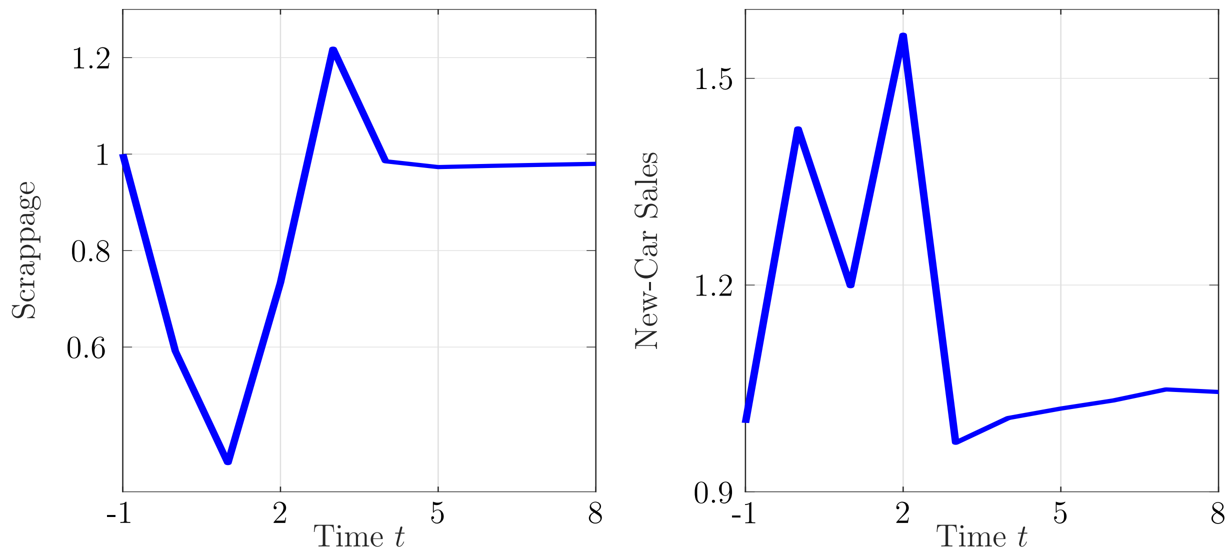

Figure B4: Credit and income shocks in the absence of secondary-market clearing. The economy is hit by the same credit and income shock as in Figure 14. The bond market clears. However, the market for used cars does not clear—that is, cars can be traded with the rest of the world at the prices prevailing in the initial stationary equilibrium. The left panel displays scrappage and the right panel the sales of new cars.

Second, we recompute the transitional dynamics with the aggregate credit shock and the aggregate income shock, clearing both credit and car markets, but setting the transaction costs equal to zero, as we did in Figure 13 for the baseline case without aggregate income shocks. Figure B5 displays these results. The dashed line represents the dynamics without transaction costs, and the solid line reproduces the dynamics obtained in Figure 14 with transaction costs. Similar to our findings of Section 6.2, the absence of transaction costs induces a spike in downgrading activity in the recession, leading to a temporary increase in the trading volume of used cars, a more sizable decline in used prices, and a larger fall in scrappage and new production, compared with the economy with transaction costs.

B.5 Skewed Income Shock

We now show that the main mechanism highlighted in the paper also arises in the presence of skewed income shocks, even without shocks to the credit supply. The empirical literature on the skewed effects of business cycles (e.g., Guvenen, Ozkan, and Song, 2014) motivates us to study the effects of a shock that decreases the income of low-income households only over a period of 2 years. We assume the income realization of low-income households (i.e., households whose income is the lowest point in our grid) decreases by 10 percent for 2 years.

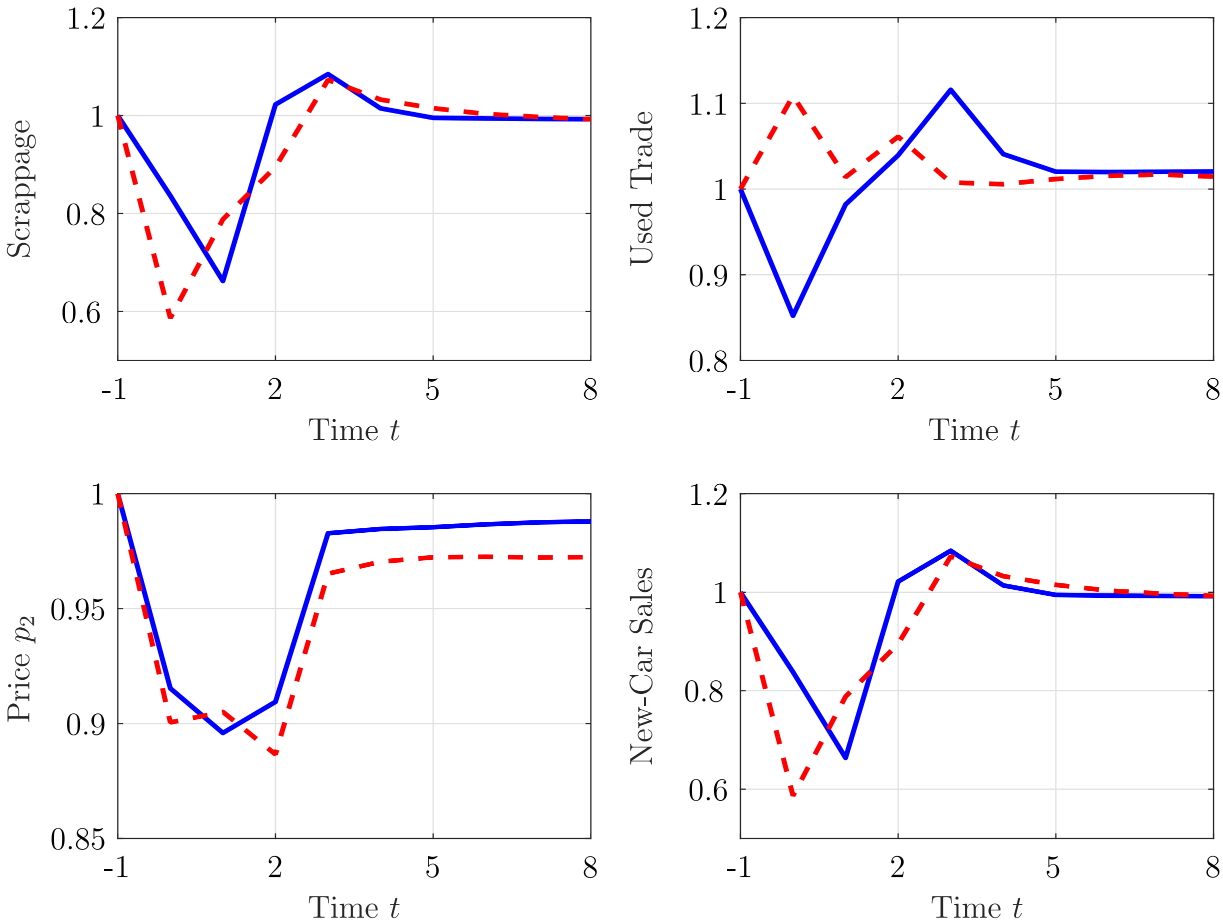

Figure B5: Credit and income shocks in the absence of transaction costs. The economy is in stationary equilibrium at $t = -1$. Households learn about the new path of the borrowing limit and income at $t = 0$. The horizontal axis displays time t. The top-left panel displays scrappage; the top-right panel the volume of trade of used cars; the bottom-left panel the price p_2 of quality-q_2 cars; and the bottom-right panel the sales of new cars. The solid line displays the case with calibrated transaction costs and the dashed line the case without transaction costs ($\lambda_0 = \lambda_1 = 0$).

The persistence of the shock over 2 years implies that this shock affects the income process of all households, either directly because of its current realization or indirectly because of the possibility of a transition to the low-income shock in the second period. For simplicity, we focus on the equilibrium in the car market and abstract from bond-market clearing, but the results are robust to general-equilibrium effects from the interest rate.

Figure B6 illustrates the effects of this shock to low-income households on the outcomes of interest. The qualitative effects are similar to those arising after the credit tightening analyzed in Section 6: Low-income households, which are temporarily hit by the income shock, choose to postpone the scrappage of their low-quality cars, inducing a decline in

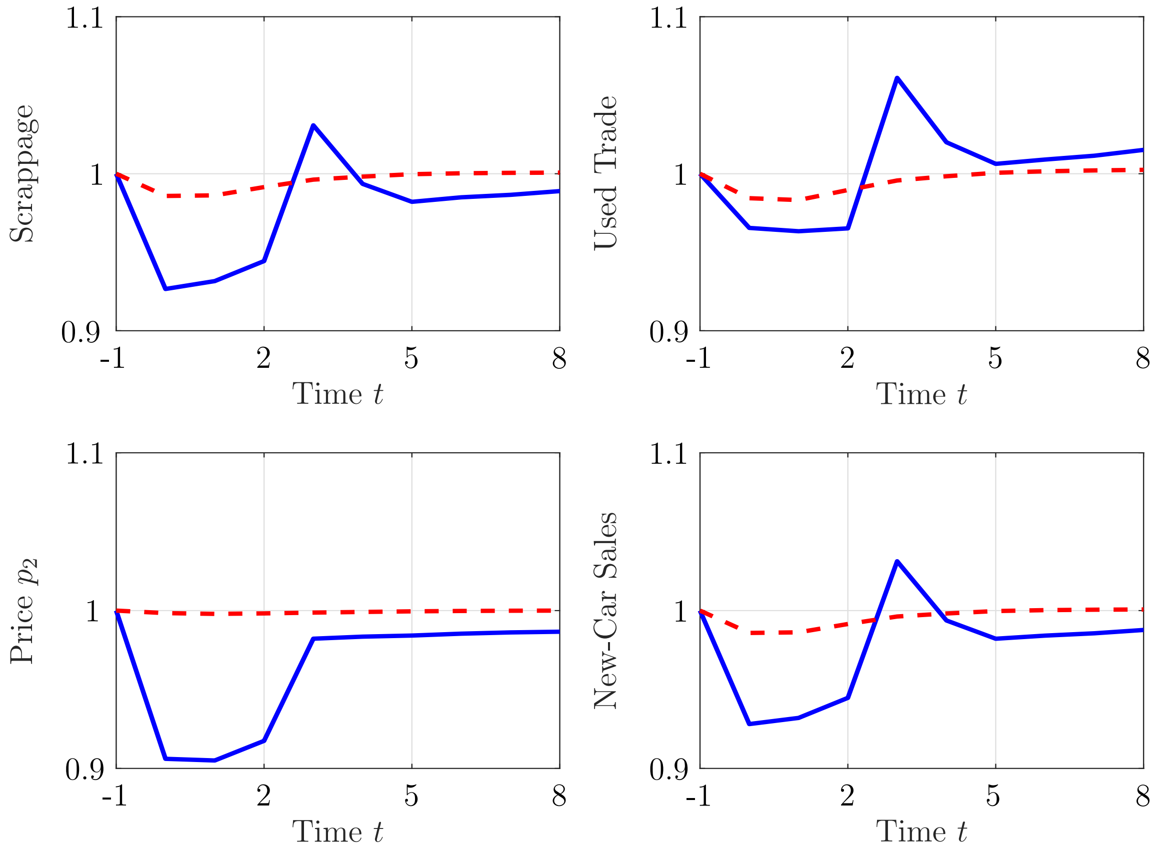

Figure B6: An income shock to low-income households. The economy is in the stationary equilibrium at $t = -1$, and households learn about the new income path at $t = 0$. The horizontal axis displays time t. The top-left panel displays scrappage; the top-right panel the volume of trade of used cars; the bottom-left panel the price p_2 of quality-q_2 cars; and the bottom-right panel the sales of new cars. The solid line displays the baseline case (credit shock), and the dashed line the case of an income shock to low-income households.

used-car prices and the volume of used-car trade; in turn, this equilibrium effect induces higher-income households to postpone the replacement of their intermediate-quality cars, leading to a decrease in new-car sales. However, the equilibrium dynamics are quantitatively smaller than those reported in Section 6: The drop in new-car sales is less than 2 percent. Hence, this analysis suggests that skewed income shocks may have contributed to the empirical patterns described in Section 3, but they are unlikely to be their main driver. Nevertheless, they could be potentially relevant to account for the dynamics of durable-goods purchases during other business-cycle episodes in which credit markets were not as affected as during the Great Recession.

C Solution Algorithm

In this appendix, we describe our algorithm to solve for the stationary equilibrium and the transitional dynamics following unexpected aggregate shocks. We emphasize our novel method to find market-clearing prices in the presence of heterogeneous agents making discrete choices, which seems applicable to a large class of models. We use this method to solve for the stationary equilibrium and the transitional dynamics of our model.

C.1 Stationary Equilibrium

We now provide the main steps in solving for the stationary equilibrium of the model (see Definition 1 in Section 4.3).

1. We discretize the set of possible states for bonds b (using a fine grid with $N_b = 600$ nodes) and income w, using the method in Rouwenhorst (1995) with three nodes.

2. We guess a bond price p_b and car prices p_n for $n = 2, ..., N - 1$.

3. We solve for the value function V on the discretized state-space by iterating on the Bellman equation (5). We let households choose bonds on a continuum by interpolating the continuation value. We obtain the policy functions g_b and g_n.

4. We compute the stationary distribution m on the discretized state-space using a non-stochastic simulation: We start from an initial distribution and then iterate on the law of motion of the distribution implied by the policy functions and the transition probabilities of the income and depreciation shocks. We allocate households to grid points for bonds according to the distance between their desired level of bonds and the two closest grid points, following the method of Young (2010).

5. We compute excess demand for bonds and cars, and update prices using a quasi-Newton method until all markets clear. We describe the details of the market-clearing procedure in subsection C.3.

C.2 Transitional Dynamics

We now provide the main steps in solving for the transitional dynamics, assuming the economy is initially in the stationary equilibrium and households learn about the new aggregate conditions at $t = 0$. To compute the equilibrium dynamics, we need to solve for sequences of prices $\{p_{b,t}, p_{n,t}\}_{t=0}^{T-1}$, policy functions $\{g_{b,t}, g_{n,t}\}_{t=0}^{T-1}$, and household distributions $\{m_t\}_{t=1}^{T-1}$ such that households maximize utility, all markets clear in each period, and the distribution evolves according to households' policy functions, to the transition probabilities of the idiosyncratic income, and to the depreciation shocks. We apply a sequential solution algorithm as described by Ríos-Rull (1998).

1. We compute both the initial and final stationary equilibria using the algorithm described above.

2. We set the number of periods (years), $T = 30$, by which we assume the economy converges to the final stationary equilibrium.

3. We guess a sequence of bond prices and car prices $\{p_{b,t}, p_{n,t}\}_{t=0}^{T-1}$ for $n = 2, ..., N-1$.

4. We initialize the algorithm parameter $S = 0$ that we use in the following steps.

5. We obtain a sequence of policy functions $\{g_{b,t}, g_{n,t}\}_{t=S+1}^{T-1}$, by iterating backward in time from $t = T-1$ to $t = S+1$ and solving the household maximization problem in each period, using interpolation of the continuation value. Notice that at $t = T-1$, the continuation value is simply given by the value function V associated with the final stationary equilibrium.

6. Taking as given all prices, decision rules, and value functions from $t = S+1$ onward, we look for the prices $p_{b,S}$ and $p_{n,S}$ and associated decision rules $g_{b,S}$ and $g_{n,S}$ such that all markets clear at $t = S$, given the distribution of households m_S. We look for market-clearing prices using a quasi-Newton method, described in more detail in subsection C.3.

7. We update the distribution of households using the obtained policy functions and compute m_{S+1}, using a nonstochastic simulation. We allocate households to grid

points for bonds according to the distance between their desired level of bonds and the two closest grid points, following Young (2010).

8. We iterate on steps 4-7 by sequentially setting $S = 1, ..., T-1$, hence clearing markets one period at a time and obtaining a new sequence of prices.

9. We compute a convex combination of the guessed price sequence and the newly obtained price sequence and restart from step 4. We continue this procedure until convergence of the price sequence.

C.3 Market-clearing Method

Our model features heterogeneous agents making a discrete choice over car quality. The discreteness of the policy functions generates a challenge in clearing markets: The excess demand function for a given car quality is a step function, leading to either inaccuracy or failure of standard root-finding methods.

To explain this problem and our proposed solution, we now use a simplified version of our model in stationary equilibrium, in which only two car qualities exist, $n = 1, 2$. Thus, we only need solve for the relative price of cars of quality q_2, $p \equiv p_2/p_1$. Car scrappage is exogenous, and so is the bond price. Moreover, let us restrict attention to heterogeneity in income w and wealth b, by assuming that all households have the same no-car utility type θ. Thus, we consider the discretized space for the state (b, w, n).

First, we introduce some convenient notation. Let us consider all households with a given income realization $\bar{w}$ that own cars of a given quality $\bar{n}$. These households differ in their wealth b, which we discretized on a grid $\{b_j\}$ for $j = 1, ..., N_b$, where j denotes a grid point.

Let $m_j(\bar{w}, \bar{n})$ be the fraction of households at grid point j at the beginning of the period. Let $b^*(\bar{w}, \bar{n}; p) \in [\phi, b_{N_b}]$ be the threshold for wealth such that only households with wealth above $b^*(\bar{w}, \bar{n}; p)$ choose a car of quality q_1, given a relative price p; that is,

$$g_n(b, \bar{w}, \bar{n}) = \begin{cases} 2 & \text{if} \quad b \leq b^*(\bar{w}, \bar{n}; p) \\ 1 & \text{if} \quad b > b^*(\bar{w}, \bar{n}; p). \end{cases} \tag{C1}$$

Notice that in general, $b^*(\bar{w}, \bar{n}; p)$ does not coincide with any grid point for b. Let b_J and b_{J+1} be the two neighboring grid points, such that $b_J < b^* < b_{J+1}$.[C1]

Total demand for cars of quality 2 coming from households with income $\bar{w}$ and car $\bar{n}$ equals $\sum_{j=1}^{J} m_j(\bar{w}, \bar{n})$; that is, the mass of households whose wealth is below the threshold. Under standard continuity properties of the value function V, the threshold is a continuous function of the price p. Hence, for small changes in p, the threshold $b^*(\bar{w}, \bar{n}; p)$ is still between the same grid points. Accordingly, no change occurs in the total quantity demanded by households with income $\bar{w}$ and car $\bar{n}$. A sufficiently large price change, instead, implies that the threshold crosses one of the closest grid points, either b_J or b_{J+1}, leading to a discrete change in the quantity demanded. This point shows that total demand conditional on a given realization of income and car quality is a step function.

Aggregate demand for cars of quality q_2 is the sum of demands from all discrete income and car-quality values. Because the sum of multiple step functions is also a step function, aggregate demand is a step function. Moreover, the total amount of cars of quality q_2 is fixed at the beginning of the period. Hence, total excess demand (demand minus supply) is also a step function with respect to the price.

Finding a zero of a step function is problematic for numerical nonlinear equation solvers. Moreover, the simple approach of stopping at a price that gives the minimum absolute excess demand can be quite inaccurate, even with a large number of grid points.[C2]

We propose an intuitive, efficient, and easily applicable solution to obtain a continuous excess demand function and achieve accuracy in market clearing. The key idea is that continuity can be achieved by making the distribution of households close to the threshold depend on the distance between the threshold and the neighboring grid points.

Specifically, we compute the threshold associated with a given guessed price. Next, we take the beginning-of-period distribution m and we move a fraction of agents from grid

[C1] In the interest of simplifying notation, we avoid explicitly expressing J as a function of $(\bar{w}, \bar{n})$, but it is understood that each income and car-quality state has associated thresholds and neighboring grid points.

[C2] In our model, this approach does not achieve a market-clearing error below 10^{-3}, even with 1,000 grid points for bonds. Furthermore, this issue cannot be easily solved by using Monte Carlo simulation instead of a nonstochastic simulation. One can use similar arguments to show that a Monte Carlo simulation also leads to an excess demand that takes the shape of a step function. Moreover, the size of the market-clearing error guaranteed by this approach equals the inverse of the number of agents used in the simulation. This relation leads to a substantially higher computational cost than our proposed method, for a given desired level of accuracy.

point J to $J+1$, proportionally to the distance between the threshold and grid point b_{J+1}:

$$m_{J\to J+1} = \frac{b_{J+1} - b^*(\bar{w}, \bar{n}; p)}{b_{J+1} - b_{J-1}} m_J. \tag{C2}$$

We rationalize this choice as follows. We interpret each mass point m_J as a discrete approximation of the true distribution of households with a level of wealth in a neighborhood of grid point b_J. We propose an alternative, continuous approximation of this distribution, which we construct by distributing households at grid point b_J over the interval $[b_{J-1}, b_{J+1}]$. If we distribute these households using a uniform distribution, a fraction $\frac{b_{J+1}-b^*(\bar{w},\bar{n};p)}{b_{J+1}-b_{J-1}}$ of households are at grid point b_J according to the discrete approximation of the distribution, but are instead to the right of the threshold $b^*(\bar{w}, \bar{n}; p)$ under the uniform approximating distribution.[C3] Hence, they should make the same car-quality choice as households at grid point b_{J+1}.

Symmetrically, we move a fraction of agents from grid point $J+1$ to J as follows:

$$m_{J+1\to J} = \frac{b^*(\bar{w}, \bar{n}; p) - b_J}{b_{J+2} - b_J} m_{J+1}. \tag{C3}$$

We get a new distribution $\tilde{m}$, which coincides with m, except at the grid points that are closest to the thresholds; in particular, $\tilde{m}_J = m_J + m_{J+1\to J}$ and $\tilde{m}_{J+1} = m_{J+1} + m_{J\to J+1}$. Next, we use the modified distribution to evaluate aggregate demand for car quality q_2. Thanks to the continuity of b^* with respect to the price, it is easy to prove that the expression $\sum_{j=1}^{J} \tilde{m}_j(\bar{w}, \bar{n})$ is a continuous function of p. Hence, total excess demand is a continuous function of the price, allowing us to find a zero with arbitrary accuracy with standard nonlinear solvers.

In the interest of consistency in the treatment of all of the markets, we also use $\tilde{m}$ to clear the bond market. Moving agents to close grid points for bonds is similar to of we deal with the discreteness of the grid and continuity of the bond policy function g_b, following the simulation method Young (2010) proposed.

Although we referred to a simplified model, the method generalizes to the richer model of Section 4. In practice, our algorithm to clear markets for both the stationary equilibrium

[C3] Alternative closed-form expressions for the mass of agents who move between grid points can be found by assuming other approximating distributions; for instance, a truncated normal. This alternative assumption leads to quantitatively negligible differences in the solution.

and the transitional dynamics works as follows:

1. We guess prices for bonds and car qualities.

2. We solve the household problem and compute all of the thresholds for wealth such that households are indifferent between any two car qualities chosen in equilibrium, for each level of income, car quality, and type.

3. We transform the distribution by shifting households close to the thresholds proportionally to the distance between the thresholds and the neighboring grid points, as in equations (C2) and (C3).

4. We use the transformed distribution to evaluate excess demand for bonds and car qualities.

5. We update prices using a quasi-Newton method until markets clear.

6. We use the obtained policy functions and transition probabilities of the idiosyncratic shocks to update the transformed distribution associated with equilibrium prices and obtain the next beginning-of-period distribution.

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